Exercise 22

Differentiate the function.

$$y = \log_2(x \log_5 x)$$

Solution

Take the derivative of the function.

$$\begin{aligned}
y' &= \frac{d}{dx}\left[\log_2(x \log_5 x)\right] \\
&= \frac{1}{(x \log_5 x)\ln 2} \cdot \frac{d}{dx}(x \log_5 x) \\
&= \frac{1}{(x \log_5 x)\ln 2} \cdot \left\{\left[\frac{d}{dx}(x)\right]\log_5 x + x\left[\frac{d}{dx}(\log_5 x)\right]\right\} \\
&= \frac{1}{(x \log_5 x)\ln 2} \cdot \left[(1)\log_5 x + x\left(\frac{1}{x \ln 5}\right)\right] \\
&= \frac{1}{x \ln 2} + \frac{1}{x \log_5 x \ln 5 \ln 2} \\
&= \frac{1}{x \ln 2} + \frac{1}{x \frac{\ln x}{\ln 5} \ln 5 \ln 2} \\
&= \frac{1}{x \ln 2} + \frac{1}{x \ln x \ln 2} \\
&= \frac{1}{x \ln 2}\left(1 + \frac{1}{\ln x}\right)
\end{aligned}$$